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A NEW METHOD FOR THE DESIGN OF RETAINING WALLS.

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To be read Thursday, 5th December, 1895.

The method proposed is to take as a basis a wall which supports an unlimited slope of loose material at its own angle of rest, and to bring all other cases into relation to this. The advantage of approaching the problem from this direction is, that the formulæ become so simple that the comparison between different forms of walls can be made with facility; and from these a wall of any required stability to meet existing conditions can be conveniently and correctly deduced.

To show at the outset that the basis proposed is not so far from natural conditions as is usually supposed, the circumstances which first directed the writer's attention to this method may be briefly mentioned. Along the Fraser and Thompson Rivers in British Columbia, on the line of the Canadian Pacific Railway, there are stretches of some miles at different places where very long slopes of sand and gravel occur. These slopes extend from near the edge of the river to a height of from 300 to 500 feet or more, where they reach the top of a terrace. They are usually more or less covered with grass and sage-brush, although some broken gaps also occur which are termed gravel-slides. The general location of the railway along the face of these slopes is at about half the height between the river and the top of the terrace above. Although this class of country has in general an irregular appearance, yet a careful examination of these long slopes shows that they are exceedingly uniform in their inclination where the material is sand and gravel; and this inclination does not vary appreciably from 33° 41′, or $1\frac{1}{2}$ to 1, the well-known angle of repose for material of this kind. There also occur long natural slopes of rock debris in the more rocky parts, which show with great uniformity an inclination of 37° to 40°, or $1\frac{1}{4}$ to 1. These are thus examples of natural "surfaces of equilibrium" such as are seldom met with on so large a scale.

These slopes, while maintaining their general inclination as the side of the main valley, are more or less fluted or corrugated on an immense scale, by the valleys of the side streams. In these circumstances the railway cannot follow a natural contour without bringing in an excessive amount of curvature; but in making a series of cuttings and embankments on this side-hill ground, a practical difficulty at once arises, as the slope of both cutting and embankment would be parallel to the natural surface for a vertical height of 200 or 300 feet both above and below.

As the writer had occasion a few years ago to make estimates for the completion on a permanent basis of the railway works on 125 miles along these rivers, it was at once evident that the construction of retaining walls to support the slopes of both cuttings and embankments would be the most economical method to pursue. In designing such walls for stability, the pressure which they have to withstand is practically the same as for an unlimited slope at the angle of repose, which shows that this condition is no mere theoretical consideration.

The complexity that attaches to the problem of the stability of retaining walls arises from the fact that authors have usually considered the case of some arbitrary limiting surface for the earth to be supported, instead of the natural surface of equilibrium. The simplification of the formulæ which results in this case is sometimes touched upon as a theoretical corollary of no practical value. But the consideration of the surface of equilibrium, inclined at its own angle of friction to the horizon, brings the question into relation to the hydrostatic problem of walls for the support of water. The simplicity in their case is due to the fact that for them a surface of equilibrium is always considered.

The use of walls to support a horizontal surface of earth on a level with the crest of the wall itself (known on railways as grade walls) is so frequent in practice as to require special consideration, and to deal with this case satisfactorily certain arbitrary simplifications have to be introduced. But for walls with any considerable surcharge, which come nearer to the natural conditions as already explained, the best method will be to design first a wall of equilibrium which is at the limit of stability when supporting an unlimited surface at the angle of repose; then to allow the difference between this and the actual surcharge as a margin of safety; and if necessary in addition, to increase the wall itself for greater stability. The wall of equilibrium can be calculated without any assumption or simplification of the actual conditions; and the margin or increase for stability which there is to count upon becomes quite distinct. It is possible also in dealing with a surface of equilibrium, to calculate the stability of dry masonry walls, or even a heavy layer of rip-rap for the support of earthwork.

The term "loose material" is preferable to "earth," as it avoids conveying the impression that there may be cohesion in the material; as any cohesion at once vitiates the theory. Cohesion, too, may not always be on the side of safety, especially in the case of water-soaked material, which by itself may possess a certain amount of cohesion, and may yet exert an increased pressure on the wall. The term "loose material" is also the most general, and may be taken to include sand and gravel, loose rock, or even corn stored in an elevator.

It is also essential to distinguish clearly between the modes in which a retaining wall may fail, in order to know definitely the conditions under which the various formulæ are applicable. These are :—

(A) Failure by overturn around the outer edge of the base. This is the mode of failure which is generally taken for granted; but authors seldom point out what is really implied by this condition. It means physically that the surface on which the wall stands is incompressible, inelastic, and in fact absolutely resisting. This may be closely true for a foundation on rock or other very hard material. But if the surface has any elasticity, it is easy to show from the theorem of the "central third," that the resultant of the reaction against the base will have its point of application at two-thirds of the width from the back of the wall, instead of at the outer edge. With ordinary forms of walls, this will decrease the stability to about one-half; and it is a question how far this should be taken into account for walls standing on pile foundations or on timber platforms, where the elasticity may be quite appreciable.

(B) Failure by sliding on the base, or at the level of any course in the masonry. This is often referred to; but is of less practical importance, because the weight of ordinary walls gives them a greater resistance by friction than by overturn; and if the friction is found deficient, it is more economical to adopt some expedient for increasing it, such as inclining the foundation courses against the direction of the pressure, rather than to increase the dimensions of the wall itself.

(C) Failure by sliding at an angle through the mass. This can only occur when the wall itself is of loose material; as, for example, loose rock supporting earthwork by its superior weight and greater angle of friction; a case which we will consider further on.

In this paper, failure by overturn in the ordinary way will be considered, unless otherwise stated.

Wall supporting an unlimited extent of loose material at the angle of repose.

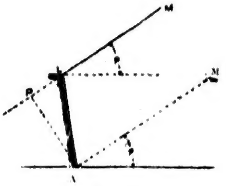


Fig. 1

Let A L represent the back of the wall, and L M′ the surface of the loose material extending to an unlimited distance at its own angle of repose ϕ. It is evident that the pressure against the wall is due entirely to the layer between the lines LM′ and AM, as the remaining material below the line AM is already in equilibrium without the aid of the wall.

In finding a formula for the pressure against the wall, we obtain the simplest expression by taking the value of the horizontal component of the total pressure, usually called the horizontal thrust; and this thrust is also the most convenient in finding the moment of overturn. In order to avoid mathematical detail, the proof of the formula is given in the Appendix. A graphical method of general application is there given, from which the proof is deduced; and the simplifications which obtain in the case considered are thus easily traced. The result is also shown to correspond with formulæ which Rankine gives for special cases.

The resulting formula is as follows:—

Let $T =$ the horizontal thrust.

$w =$ the weight of the loose material supported, per unit of volume.

$p =$ the perpendicular distance A P from the foot of the wall to the surface of the loose material; *i.e.*, the thickness of the layer supported.

$$\text{Then } T = \tfrac{1}{2} w p^2$$

It will be noted in the proof that the elimination of other variables enables the following statement to be made:—

When a wall supports an unlimited surface of loose material at an angle of repose, the horizontal component of the pressure on the wall is independent of the existence or non-existence of friction between the loose material and the wall. Its amount depends only upon the weight of the loose material per unit of volume, and upon the perpendicular thickness of the layer supported. It is also independent of the inclination of the wall, provided that the thickness of the layer supported is unchanged. Its point of application is at one-third of the vertical height of the wall.

Thus in the following figures, in which the thickness p of the layer supported is the same, the absolute value of the thrust T will be the same for the various inclinations of the back of the wall; but the point of application will be at a variable height above the *horizontal* plane through A, and the moment of overturn will increase accordingly.

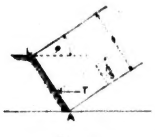


Fig. 2

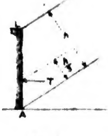


Fig. 3

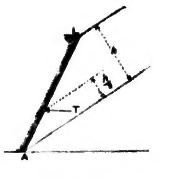


Fig. 4

The identity of the formula here found, with the ordinary formula for the hydrostatic pressure of a liquid, is apparent. We find therefore in this case, that the horizontal thrust due to the pressure of any class of loose material is the same as that of a liquid which has the same weight per cubic foot, provided that the thickness of the layer supported is the same as the vertical depth of the liquid. The only physical difference is that the loose material must have a surface of unlimited extent, while the pressure of the liquid is the same with a limited surface area.

Walls of Equilibrium.

We may now proceed to investigate the most economical forms of "walls of equilibrium," or walls which are on the point of overturning when supporting loose material of unlimited extent at its angle of repose. It is evident that a triangular form is the most economical. (As ALC in Figs. 21 and 22.) In the case of a rectangular wall for an infinite surcharge, in the series recommended by Poncelet, the base is 0.934 of the height, while the stability is only 1.86, which shows at once how far such a section is from being economical. If in Figs. 21 and 22 we adopt any given face batter (angle δ), and proceed to calculate the required back batter (angle θ), the general formula is a quadratic in terms of $\tan\theta$, which we give with proof in the Appendix (IV). The reason of the degree of complexity which this formula presents is that the thickness p of the layer supported by the wall is itself a variable as θ varies.

The forms in Figs. 5 and 6 are calculated by this general formula, and they represent two typical cases of sand and gravel supported by masonry walls :—

Firstly, an unlimited slope EF at $1\frac{1}{2}$ to 1 at a vertical height AB_1 above the point A. This is a very usual case in practice, as the point A may be the outer edge of foundation on side-hill ground, or the inner edge of a railway formation or road-bed, and the slope EF has to be supported at some point between B_1 and B_5. For comparison the height AB_1 is taken as 10 units, and the areas of the cross-sections are as follows :—

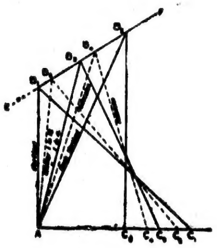


Fig. 5

Height $AB_1 = 10.00$

Area	AB_1C_1	= 56.50
"	AB_2C_2	= 55.38
"	AB_3C_3	= 53.10
"	AB_4C_4	= 51.64
"	AB_5C_5	= 47.02

Secondly, the wall to be built on a natural surface GH at $1\frac{1}{2}$ to 1, and to support an unlimited slope EF at a height AB above A. Here EF may be taken to represent the slope of an embankment on side-hill ground.

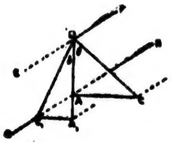


Fig. 6

Height AB = 10.00

Area	ABC	= 56.50
"	A_1BC_1	= 47·02

In both these cases the wall with a vertical back is the most economical, and it also requires less excavation for its foundation.

The following table of triangular walls of equilibrium shows the values required for the face and back batters, in some of the more

usual cases. The rip-rap is considered to be in equilibrium around the outer edge of the base as in the case of masonry, and the negative sign indicates that the back batter is on the same side of the vertical as the face batter. Isosceles walls are indicated by the equality of the two batters.

Conditions. – Angle $\phi = 33° \ 41'$, or $1\frac{1}{2}$ to 1. Weight w of earth = 100 lbs. per cubic foot. Weight W of masonry, etc., = 167 or 125 lbs., as stated. Calculated by the general formula in Appendix (IV).

Numerical Values for Walls of Equilibrium.

Material of wall.	$\frac{w}{W}$	Tan δ — Face batter.	Tan θ — Back batter.	Ratio of base of wall to height.
Masonry.	$\frac{100}{167} = \frac{3}{5}$	Vertical	1.1300	1.1300
		$\frac{1}{12} = 0.0833$	0.9123	0.9956
		$\frac{1}{4} = 0.2500$	0.4885	0.7385
		0.3191	0.3191	0.6382
		0.4558	Vertical	0.4558
Dry Masonry.	$\frac{100}{125} = \frac{4}{5}$	0.3810	0.3810	0.7620
		$\frac{1}{2} = 0.5000$	0.0665	0.5665
		0.5263	Vertical	0.5263
Rip-Rap.	$\frac{100}{125} = \frac{4}{5}$	$\frac{1}{1} = 1.0000$	-0.9240	0.0760
		0.8366	$-\frac{2}{3} = 0.66$	0.1700

The above table may be extended by means of the general formula to include any class of loose material, supported by walls of any given weight per cubic foot. The formulæ for walls with either the face or the back vertical, deduced from the general formula, are also given in the Appendix. If preferred, the forms of such walls of equilibrium may be found from the original formula $T = \frac{1}{2} w p^2$ and the moment of stability, by means of direct trial and successive approximations.

We may also obtain a comparison of the pressures exerted by different classes of loose material, by means of a ratio between them. Let the two classes be defined as follows :—

T_1 = horizontal thrust ; w_1 = weight; ϕ_1 = angle of repose
T_2 = " " w_2 = " ϕ_2 = " "

$$\text{Then} \quad \frac{T_2}{T_1} = \frac{w_2 \cos^2 \phi_2}{w_1 \cos^2 \phi}$$

Numerical examples. $\phi_1 = 33° \ 41'$ for sand, and $\phi_2 = 45°$ for loose rock. Then :—

$$\frac{T_2}{T_1} = \frac{w_2}{w_1} \ \frac{13}{18}$$

Also, if the stone weighs 167 lbs. per cubic foot, the broken stone or loose rock formed from it will have 40 per cent. of void, and will weigh 100 lbs. per cubic foot, which is the same as for sand. Hence $w_2 = w_1$, and we have

$$T_2 = 0{\cdot}722 \ T_1$$

These ratios, whether algebraical or numerical, will be closely approximate for any considerable height of surcharge.

Grade Walls, or walls supporting loose material limited by a horizontal surface at the level of the crest of the wall.

Here we have the loose material limited by an arbitrary surface; and the element of friction between it and the wall is no longer eliminated as in the case of a surface at repose. Hence there are two conditions or assumptions to be clearly distinguished :—

(1) The existence of friction between the loose material and the back of the wall, equivalent in amount to the angle of friction ψ. In this case the best solution is the graphical one, as shown in Figure 15 in

the Appendix, the surface line being made horizontal. The algebraical method is complicated; although some simplification is obtained by assuming $\psi = \phi$, the angle of friction for the material itself, which is usually a close approximation.

(2) No friction between the loose material and the wall. This is clearly an assumption; but it is generally employed in practice, because it gives a greater value for the thrust than when the friction is taken into account, and therefore leaves a greater margin on the side of stability. This additional stability may also be taken to make up for the effect of vibration occasioned by either a train or waggon where the wall supports a road-bed.

The graphical and algebraical methods for this case are given in the Appendix (III), and also the proof of the following formula for the horizontal thrust T:—

$$T = \frac{wh^2}{2} \cdot \frac{1 - \sin(\phi - \epsilon)}{1 + \sin(\phi + \epsilon)}$$

(The sign of ϵ will be reversed if the inclination ϵ is on the other side vertical. See Fig. 20.)

We can obtain an interesting comparison between the amount of pressure occasioned by the same kind of material in this case and in the original case of an unlimited slope at repose, if we simplify the formulæ to represent a wall with the back vertical.

Let T_1 = horizontal thrust for an unlimited slope at the angle of repose ϕ, as already found.

T_2 = horizontal thrust for a horizontal surface.

h = vertical height of the wall.

$$T_1 = \tfrac{1}{2} w p^2 = \tfrac{1}{2} w h^2 \cos^2 \phi$$

$$T_2 = \frac{w h^2}{2} \cdot \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{w h^2}{2} \cdot \tan^2 \tfrac{1}{2}(90 - \phi)$$

$$\frac{T_2}{T_1} = \frac{\tan^2 \tfrac{1}{2}(90^\circ - \phi)}{\cos^2 \phi} = \frac{1}{(1 + \sin\phi)^2}$$

The following numerical values from this formula will serve as illustrations of the ratios obtained:—

For	$\phi = 30^\circ\ 00'$	$T_2 = 0.4444\ T_1$
"	$\phi = 33^\circ\ 41'$	$T_2 = 0.4137\ T_1$
"	$\phi = 38^\circ\ 39'$	$T_2 = 0.3788\ T_1$
"	$\phi = 45^\circ\ 00'$	$T_2 = 0.3431\ T_1$

From such ratios we can obtain a decided advantage in simplicity of method; for we may first determine the best form of wall for an unlimited slope of the class of loose material in question, and then find its factor of safety when the material is cut down to a horizontal surface. If this is still insufficient, an additional amount can be added to the cross-section to bring the safety up to any required factor.

Walls of Equilibrium and actual Cross-Sections.—After determining the wall of equilibrium ABC for any conditions, by the methods described, an addition can be readily made to the cross-section to give the wall any desired stability, as shown in the shaded portions of the figures. As the pressure against the wall is unchanged by this addition, the factor of safety will be the ratio of the moment of stability of the completed wall to the moment of stability of the wall of equilibrium. An advantage in this method is also that the actual margin of safety is apparent.

Fig. 7

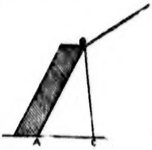


Fig. 8

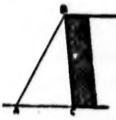


Fig. 9

Fig. 10

Loose Rock supporting Earth.—Failure of the supporting mass by sliding at an angle, as described at first as mode (C).

The mode of failure assumed so far has been that the wall will turn around the outer edge of the base, and fail by upsetting. This implies that the foundation is unyielding, and also that the friction on the base is sufficient to prevent the wall from pushing forward horizontally. These modes of failure have been explained at the outset and indicated by the letters (A) and (B). We will now refer to the third mode of failure, in which the supporting mass may give way by sliding at an angle on itself. This may occur when rip-rap or loose rock is used for the support of earthwork. In the region referred to, there were two problems of this kind. (1) A side-hill embankment made of loose rock on the outside. (2) A cutting with the slope supported by rip-rap.

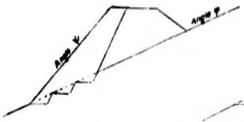


Fig. 11

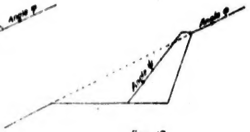


Fig. 12

These conditions are illustrated by the figures, and the question is the thickness of rock embankment or rip-rap required. As the supporting material is itself loose, the support it gives will be due to its weight and friction only; and it will fail by shearing or sliding along some line of least resistance. The position of this line in the mass involves the determination of a minimum under conditions which may be thus stated:—

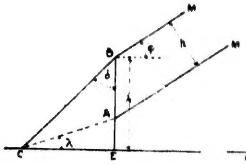


Fig. 13

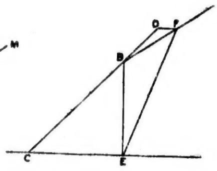


Fig. 14

Let CBE be the mass which supports loose material with a surface BM′ at the angle of repose. The line of least resistance CA, or line of shear along which the mass will give way by sliding, will be inclined at some angle λ to the horizon. Required the value of λ which will give a minimum value to this resistance; and the angle δ which will give sufficient thickness to insure stability. It is to be noted that the thickness p of the layer supported varies also with the angle λ. The simple expression already found for the horizontal thrust T of this layer enables a relation between the angles δ, λ, and the angles of friction ϕ and ψ to be found, as given in the Appendix (V).

For the most important practical case of loose rock supporting sand and gravel, the weight of the two kinds of material per cubic foot is practically the same; and the stability depends entirely on the increased angle of friction in the loose rock, the angles being $\phi = 33^\circ 41'$

and $\psi = 45°$ respectively. If we make also $\delta = 45°$ there will b equilibrium in the lower part of the mass CBE up to the inclination $\lambda = \phi$; but above this the top of the triangle will be pushed off by the pressure. There will be a value of λ which gives a minimum, however, if δ is greater than $45°$, and equilibrium will then obtain. In practice, the upper part of the triangle can be further strengthened by adding the area BDFE.

For the support of a horizontal surface, as in the case of embankment Fig. 11, this solution is sufficiently close, as the relatively low cost of loose rock, even when it has to be obtained by excavation, makes it allowable to use an amount considerably in excess of theoretical requirements. It also makes an embankment practicable on steep side-hill ground, and where a masonry wall would be much more expensive.

In the case of a cutting, Fig. 12, the reverse is usually the case, because the volume of masonry required is so much less than the amount of loose rock laid on as rip-rap, that at ordinary prices the masonry is the more economical of the two. This is probably the reason that this problem has not received more attention; although there are many cases in which it is of practical value.

APPENDIX.

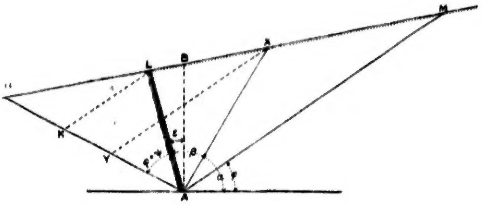


Fig. 15

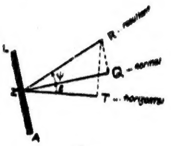


Fig. 16

Graphical method of determining the thrust of loose material limited by any plane LM, against the back of a wall AL.

Let ϕ = angle of repose; *i.e.* the angle of friction of the loose material on itself.

ψ = angle of friction of the loose material against the wall.

ϵ = inclination of back of wall to the vertical.

w = weight of loose material per unit of volume.

Q = pressure normal to the wall.

R = resultant pressure at an angle ψ to the wall.

T = horizontal component of R.

The relation of Q, R and T is shown in the lower figure.

I. *General Method.*—Applicable to all inclinations of the surface and the wall, and to all classes of loose material.

Draw AM at an angle ϕ with the horizontal.

Draw AO at an angle $\phi + \psi$ with AL.

Draw LK parallel to AM; and take OY a mean proportional between OK and OA; and draw YX parallel to AM. Then XA will be the line limiting the prism of maximum pressure.

Also the general equation giving the value of the pressure is:—

$$\frac{Q}{\cos\psi} = \frac{w}{2} \cdot \sin(\alpha + \beta + \psi)\ \overline{AY}^2$$

or $\dfrac{Q}{\cos\psi} = \dfrac{w}{2} \cdot \cos(\psi + \epsilon)\ \overline{AY}^2$

and also since $R \cos \psi = Q$ and $T = R \cos(\psi + \epsilon)$

we have $T = \dfrac{w}{2} \cdot \cos^2(\psi + \epsilon)\ \overline{AY}^2$

(If ϵ is on the other side of the vertical, its sign will be negative.)

For proof of this method, and the resulting general equation, see Collignon, "*Mécanique Appliquée*," Book VIII, chap. IV.

To show how general this method is, the following additional examples may be given. In the second case the point J is found by making LJ parallel to AH, in order that the triangles ALH and AJH may be equal in area, for compensation.

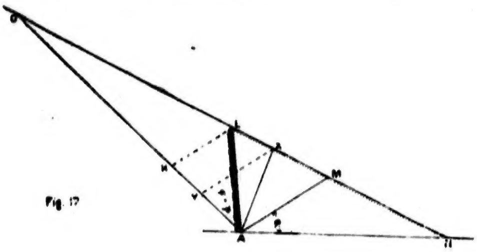


Fig. 17

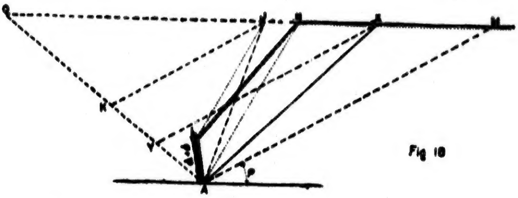
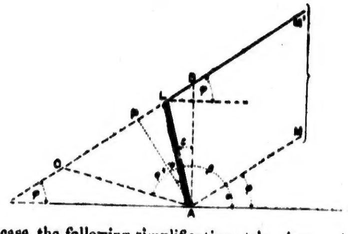


Fig. 18

II. *For an unlimited surface at the angle of repose.*



In this case the following simplifications take place. As LM′ is at the angle of repose, and therefore parallel to AM, the points K and Y coincide with O; and the length AY becomes AO. Also, since AX coincides with AM, the angle a becomes the angle ϕ; and the equation becomes:—

$$\frac{Q}{\cos\psi} = \frac{w}{2} \cdot \sin(\beta + \phi + \psi)\ \overline{OA}^2$$

or $$\frac{Q}{\cos\psi} = \frac{w}{2} \cdot \cos(\psi + \epsilon)\ \overline{OA}^2$$

and $$T = \frac{w}{2} \cdot \cos^2(\psi + \epsilon)\ \overline{OA}^2$$

Let p = the perpendicular distance AP; then as

$p = \sin(\beta + \phi + \psi)$ OA $= \cos(\psi + \epsilon)$ OA,

we have finally, $T = \frac{1}{2} w p^2$

NOTE.—In Rankine's "Civil Engineering," a formula corresponding to this is given for the special case of a wall with a vertical back. In Article 183, IV., the pressure parallel to the surface is denoted by P' and for a surface sloping at the angle of repose he gives (in our notation) $P' = \frac{1}{2}\, w \cos\phi\, \overline{AB}^2$. As $P' \cos\phi = T$, and $AB \cos\phi = p$, the formulæ are evidently identical.

III. *Surface horizontal; as for Grade Walls.*

Condition; no friction between loose material and wall. With this condition, Prôny has shown for a vertical wall, and Français for an inclined wall, that the line AX, limiting the prism of maximum pressure, will bisect the angle LAM.

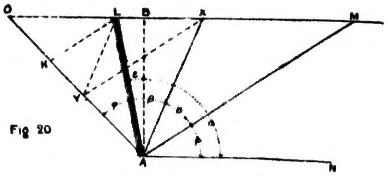


Fig 20

As the general method is also applicable to a horizontal surface, we obtain the formulæ at once by introducing the new conditions. Using the same notation as before, and making the vertical height $AB = h$, we have in this case $\psi = o$; and the normal pressure Q therefore coincides with the resultant R.

$$\text{Hence } P = Q = \frac{w}{2} \cos\epsilon\ \overline{AY}^2$$

and since we now have $T = R \cos\epsilon$

$$T = \frac{w}{2} \cos^2\epsilon\ \overline{AY}^2$$

The value of AY can be found, and the equation reduced to the algebraic form thus:—

Since AX bisects LAM, it can be shown that $OY = OL$; and therefore in the triangle OAL:—

$$AY.\ \cos\epsilon = OA\ (\cos\epsilon - \sin\phi)$$

$$\text{Also } OA.\ \cos(\phi + \epsilon) = AB = h$$

$$\text{Hence } T = \frac{wh^2}{2} \cdot \frac{(\cos\epsilon - \sin\phi)^2}{\cos^2(\phi + \epsilon)}$$

From the evident relations between the angles this becomes

$$T = \frac{wh^2}{2} \cdot \frac{1 - \sin(\phi - \epsilon)}{1 + \sin(\phi + \epsilon)}$$

(If ϵ is on the other side of the vertical AB, its sign in the above equation will become reversed.)

NOTE.—In the special case of a wall with a vertical back $\epsilon = o$, and this formula then becomes identical with Rankine's. See "Civil Engineering," Article 183, IV.

IV. *Walls of Equilibrium, general formula.*

Fig. 21

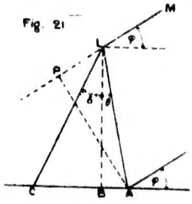
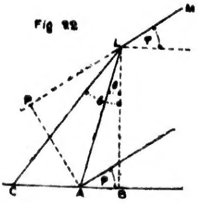


Fig 22



Method of determining the form of a triangular wall ALC, supporting the pressure of loose material at its own angle of repose LM, so that the moment of stability of the wall may be equal to the moment of overturn caused by the pressure against the back of the wall AL, the moments being taken around the point C.

Let w = weight of loose material per unit of volume.

W = weight of material of the wall per unit of volume.

ϕ = angle of repose of the loose material.

δ = angle of the face batter; and $\tan \delta = n$.

θ = angle of the back batter.

h = vertical height LB.

T = horizontal component of the pressure.

In solving the problem, either the back batter θ or the face batter δ must be assumed, and the other calculated from it.

The weight of the wall made up by the sum or difference of the two triangles CLB and ALB is:—

$$\tfrac{1}{2} Wh^2 \tan \delta \pm \tfrac{1}{2} Wh^2 \tan \theta$$

The moment of stability around the point C is:—

$$\frac{Wh^3}{6} (2 n^2 + 3 n \tan \theta + \tan^2 \theta)$$

The moment of overturn around C is:—

$$\frac{h}{3} \cdot T = \frac{wh^3}{6} (\cos \phi \pm \sin \phi \tan \theta)^2$$

By equating these moments, the two following forms are obtained according as the equation is arranged to solve for n or for $\tan \theta$;

$$2 n^2 \pm 3 \tan \theta . n + \tan^2 \theta = \frac{w}{W} (\cos \phi \pm \sin \phi \tan \theta)^2$$

$$\text{or} \left(1 - \frac{w}{W} \sin^2 \phi\right) \tan^2 \theta \pm \left(3 n - \frac{w}{W} \cdot 2 \sin \phi \cos \phi\right) \tan \theta$$

$$+ \left(2 n^2 - \frac{w}{W} \cos^2 \phi\right) = 0$$

(The negative sign must be used when θ is on the left of the vertical in the figure; and in this case also the negative value of the square root must be taken in solving the equation for $\tan \theta$.)

When the back batter is vertical, $\theta = 0$

$$\text{and also } n^2 \text{ or } \tan^2 \delta = \tfrac{1}{2} \cdot \frac{w}{W} \cdot \cos^2 \phi$$

When the face batter is vertical, $\delta = 0$, *i.e.* $n = 0$ and

$$\left(1 - \frac{w}{W} \sin^2 \phi\right) \tan^2 \theta . - \left(\frac{w}{W} \cdot 2 \sin \phi \cos \phi\right) \tan \theta$$

$$- \frac{w}{W} \cos^2 \phi = 0$$

V. *Loose Rock supporting Earth*, or other loose material. (See Figure 13.)

A relation between the angles in this case can best be obtained by finding a ratio between the pressure of the layer p and the frictional support afforded by the mass ABC along the line AC. The maximum value of this ratio will correspond with the line of minimum stability.

Let Q = weight of the triangle ABC.

T = horizontal thrust of the layer p against AB.

ϕ = angle of repose of the loose material.

w = weight per unit of volume of the loose material.

ψ = angle of friction of the supporting material ABC.

W = weight per unit of volume, of the above.

Then as the back AB is vertical,

$$Q = \tfrac{1}{2}\, Wh^2\,(1 - \tan\delta \tan\lambda) \tan\delta$$

Inclined friction along AC $= Q\,(\tan\psi - \tan\lambda)\cos\lambda$

Hor'l. component of this friction $= Q\,(\tan\psi - \tan\lambda)\cos\lambda$

Hor'l. thrust of layer $p = \tfrac{1}{2}\, w\, p_2$

$$= \tfrac{1}{2}\, w\, h^2\,(1 - \tan\delta \tan\lambda)_2 \cos_2\phi$$

Hence for ratio of hor'l. thrust to hor'l. component of friction :—

$$\frac{\text{Thrust}}{\text{Friction}} = \frac{w}{W} \cdot \frac{\cot\delta - \tan\lambda}{\tan\psi - \tan\lambda} \cdot \frac{\cos^2\phi}{\cos^2\lambda}$$

For stability this ratio must not be less than unity ; *i.e.* the angle δ must be sufficiently large to prevent the ratio from falling below unity for any value of λ between $\lambda = o$ and $\lambda = 90^\circ - \delta$.